Simulation

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Lecture Outline

- What is simulation?
- Why simulation to OR problems?
- Random number generation
- Monte Carlo simulation
- Areas of simulation application

What is Simulation?

- Mathematical and computer modeling technique for replicating real-world problem situations
- Modeling approach primarily used to analyze probabilistic problems
- The imitation of the operation of a real-world process or system *over time…*
  - Most widely used tool for decision making
  - Usually on a computer with appropriate software
  - An analysis (descriptive) tool – can answer what if questions
  - A synthesis (prescriptive) tool – if complemented by other tools
- Applied to complex systems that are impossible to solve mathematically
- Simulation does not normally provide a solution. It provides information that is used to make a decision

Simulation: A Brief Definition

Simulation is defined to be a method that utilizes sequences of random numbers as data. It is an extremely useful tool in situations where no closed form expression of a system is available or too complex to get.

Why Simulation?

- No closed form expression of the physical system
- Not possible to find an analytic solution
- Actual observation of a system is expensive and time consuming

Applications

Manufacturing facility
Bank operation
Airport operations (passengers, security, planes, crews, baggage)
Transportation/logistics/distribution operation
Hospital facilities (emergency room, operating room, admissions)
Computer network
Business process (insurance office)
Chemical plant
Fast-food restaurant
Supermarket
Theme park

What is a System?

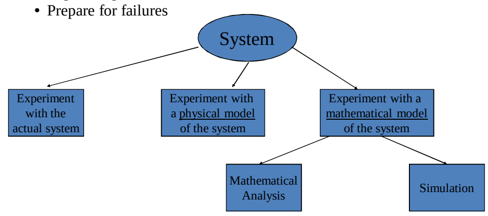
- A set of interacting components or entities operating together to achieve a common goal or objective.
- Examples
  - A manufacturing system with its machine centres, inventories, production schedule, items produced.
  - A telecommunication system with its messages, communication network servers.
  - A theme park with rides, workers, etc.

7

Why & How to Study a System

Why?

- Measure/estimate performance
- Improve operation
- Prepare for failures



8

Mathematical Model

- An abstract and simplified representation of a system
- Specifies
  - Important components
  - Assumptions/approximations about how the system works
- Not an exact re-creation of the original system!
- If model is simple enough, study it with Queuing Theory, Linear Programming, Differential Equations, etc.
- If model is complex, Simulation is the only way!

9

Getting Answers From Models

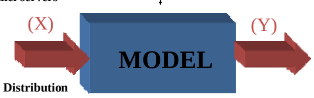
ACTUAL SYSTEM

Operating Policies

- **Single queue, parallel servers**
- **FIFO**

Input Parameters

- **No of servers**
- **Inter-arrival Time Distribution**
- **Service Time Distributions**



Output Parameters

- **Waiting Times**
- **System Size**
- **Utilizations**

Y = f (X)

10

Simulation Models

Static (Monte Carlo)

Represents the system at a particular point in time

IID observations

- Estimation of π
- Risk Analysis in Business

Dynamic Systems

Represents the system behaviour over time

Continuous Simulation:

- (Stochastic) Differential Equations
- Water Level in a Dam

Discrete Event Simulation:

- System quantities (state variables) change with events
- Queuing Systems
- Inventory Systems

11

How to Simulate?

- By hand
  - Buffon Needle and Cross Experiments
- Spreadsheets
- Programming in General Purpose Languages
  - Java, C, C++
- Simulation Languages
  - SIMAN
- Simulation Packages
  - Arena

Issue: Modeling Flexibility vs. Ease of Use

12

Advantages of Simulation

- When mathematical analysis methods are not available, simulation may be the only investigation tool
- When mathematical analysis methods are available, but are so complex that simulation may provide a simpler solution
- Allows comparisons of alternative designs or alternative operating policies
- Allows time compression or expansion

13

Disadvantages of Simulation

- For a stochastic model, simulation estimates the output while an analytical solution, if available, produces the exact output
- Often expensive and time consuming to develop
- An invalid model may result with confidence in wrong results.

14

Example

Customers arrive at a milk booth for requires service. Assume that inter-arrival time and service time are constant and given by 1.8 and 4 time units, respectively. Simulate the system by hand computations for 14 time units. Assuming that the system starts at time t = 0,

(i) find the average waiting time per customer.

(ii) find the percentage idle time of the facility?

Solution

A : event of arrival of one customer

D : event of departure of one customer

- Since the system starts at time t = 0, the first customer avails service without any delay.
- The next customer arrives into the system at time t = 0 + 1.8 = 1.8
- The first customer departs the system t = 0 + 4 = 4

We present the system in the form of a table.

Solution

Time	Event	Customer	Waiting Time
0.0	A	1	
1.8	A	2	
3.6	A	3	
4.0	D	1	4 – 1.8 = 2.2 (Customer 2)
5.4	A	4	
7.2	A	5	
8.0	D	2	8 – 3.6 = 4.4 (Customer 3)
9.0	A	6	
10.8	A	7	
12.0	D	3	12 – 5.4 = 6.6 (Customer 4)
12.6	A	8	
14.0	END	--	14 – 7.2 = 6.8 (Customer 5) 14 – 9.0 = 5.0 (Customer 6) 14 – 10.8 = 3.2 (Customer 7) 14 – 13.6 = 0.4 (Customer 8)

Solution

(i) Average waiting time per customer
= (2.2 + 4.4 + 6.6 + 6.8 + 5.0 + 3.2 + 0.4) / 8 = 3.57

(ii) Percentage idle time of the facility = 0 %

Example

An electronics shop sales different brands of Laptops and also provides repair service under warranty period. The weekly repair of Laptops follow the following frequency distribution:

Demand (x): 0 1 2 3 4

Frequency: 20 40 20 10 10

Using the following random numbers

39	73	72	75	37
02	87	98	10	47
93	21	95	97	69

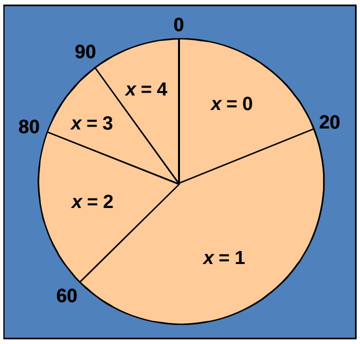
estimate the average number of weekly repair of Laptops.

Repair Distribution

LAPTOPS REPAIRED PER WEEK, *x*	FREQUENCY OF REPAIR	PROBABILITY OF REPAIR, *P*(*x*)
0	20	0.20
1	40	0.40
2	20	0.20
3	10	0.10
4	10	0.10
	100	1.00

20

Roulette Wheel of Repair



21

Generating Number of Repair from Random Numbers

DEMAND, *x*	RANGES OF RANDOM NUMBERS, *r*	
0	0-19	
1 ←	20-59	→ *r* = 39
2	60-79	
3	80-89	
4	90-99	

22

15 Weeks of Repair

WEEK	*r*	DEMAND (*x*)
1	39	1
2	73	2
3	72	2
4	75	2
5	37	1
6	02	0
7	87	3
8	98	4
9	10	0
10	47	1
11	93	4
12	21	1
13	95	4
14	97	4
15	69	2
		Σ = 31

Average of Laptops repaired
= 31/15
= 2.07 laptops/week

23

Computing Expected Number of Repair

E(*x*) = (0.20)(0) + (0.40)(1) + (0.20)(2)
+ (0.10)(3) + (0.10)(4)
= 1.5 laptops per week

Difference between 1.5 and 2.07 is due to small number of periods analyzed (only 15 weeks)

Steady-state result

24