Replacement Problems

RK Jana

Introduction

- Machines, equipments, parts lose efficiency
- Planned replacement would reduce maintenance cost and other overhead expenses
- The problem is to find the age at which it is most economical to replace it
- Certain items/parts of items fail suddenly (radio, TV, bulb)
- Immediate replacement

Two Types of Problems

- Replacement of items which deteriorate and whose maintenance cost increases with time
- Replacement of items which fail all of a sudden

Case I: Maintenance Cost Increases with Time

- **Assumption: Value of money remains same during the period.**

C: capital cost of the machine

S(t): Scrap value of the machine after t years

f(t): maintenance cost of the machine at time t

n: number of years

Annual cost of the machine at time t: C + f(t) – S(t)

Total maintenance cost: $\sum_{t=0}^{n} f(t)$

Continued…

- Total cost after n years: $T = C - S(t) + \sum_{t=0}^{n} f(t)$
- Average annual cost: $T_A = \frac{1}{n}\left[C - S(t) + \sum_{t=0}^{n} f(t)\right]$
- Replace at the end of n-th year when T_A is minimum

Example 1

A machine costs Rs 12,200. The scrap value is Rs 200. The maintenance costs of the machine are given below:

Year	1	2	3	4	5	6	7	8
Maintenance cost	200	500	800	1200	1800	2500	3200	4000

When should the machine be replaced?

$C = 12{,}200$

$S(t) = 200,$

$T_A = \frac{C - S(t) + \Sigma f(t)}{n}$

Year	$f(t)$	$\Sigma f(t)$	$S(t)$	$C - S(t)$	T	T_A
1	200	200	200	12000	12200	12200
2	500	700	200	12000	12700	6350
3	800	1500	200	12000	13500	4500
4	1200	2700	200	12000	14700	3675
5	1800	4500	200	12000	16500	3300
6	2500	7000	200	12000	19000	3167
7	3200	10200	200	12000	22200	3171
8	4000	14200	200	12000	26200	3275

T_A is minimum at the end of 6-th year. Hence it is profitable to replace the machine At the end of 6-th year

Example 2

The maintenance cost and the resale price of a truck are given below:

Year	1	2	3	4	5	6	7	8
Maintenance cost	1000	1300	1700	2200	2900	3800	4800	6000
Resale price	4000	2000	1200	600	500	400	400	400

The purchase price of the truck is Rs 8000. Determine the time at which it is profitable to replace the truck.

$C = 8000$

$T = C - S(t) + \Sigma f(t)$

$T_A = \frac{T}{n}$

Year	$f(t)$	$\Sigma f(t)$	$S(t)$	$C - S(t)$	T	T_A
1	1000	1000	4000	4000	5000	5000
2	1300	2300	2000	6000	8300	4150
3	1700	4000	1200	6800	10800	3600
4	2200	6200	600	7400	13600	3400
5	2900	9100	500	7500	16600	**3320**
6	3800	12900	400	7600	20500	3417
7	4800	17700	400	7600	25300	3614
8	6000	23700	400	7600	31300	3913

T_A is minimum at the end of 5-th year. Hence it is profitable to replace the machine At the end of 5-th year

7

Replacement of Items that Fail Completely

- There are many situations in which items do not deteriorate with time but fail all on a sudden completely. It may not be possible to predict the time of failure. Hence we make use of the probability distribution of the failure time which is obtained from past experience.
- In such situations, following two types of replacement policies are used:
  - Individual replacement policy (item replaced immediately after its failure)
  - Group replacement policy (all items are collectively replaced after a specific time period irrespective of whether they have failed or not)

8

Group Replacement Policy

In this case the total cost is calculated using

- Probability of failure at time t
- Number of items failing during time t
- Cost of group replacement
- Cost of individual replacement

9

Notations

- p(x): probability that an item will fail at the age of x
- E(x): Average failure age
- N: Number of items in the group
- C_1 : per unit replacement cost
- C_2 : individual replacement cost
- f(i): number of failure
- C(t): total cost

$$E(x) = \sum_{x=1}^{k} x\,p\,(x)$$

- Number of failure per unit $\frac{N}{E(x)}$

Thus group replacement must be made at the end of period t if the cost of individual replacement for the t^{th} period is greater than the average cost till the end of t periods. If the cost of individual replacement at the end of $(t-1)^{th}$ period is less than the average cost for t periods group replacement should not be done. In general we take one period of time as one year.

10

Example 3

The probability distribution of the failure time of a certain type of electric bulb is given below:

Week	: 1	2	3	4	5	6	7	8
FailureProb: (cumulative)	.05	.13	.25	.43	.68	.88	.96	1.0

The cost of individual replacement is Rs 4 per bulb. The cost of group replacement is Rs 1 per bulb. If there are 1000 bulbs in use then find the optimal replacement policy under

(i) individual replacement

(ii) group replacement

11

Solution

- (i) Expected life $= 1(0.05) + 2\,(0.08) + 3\,(0.12) + 4(0.18) + 5\,(0.25) + 6\,(0.20) + 7(0.08) + 8\,(0.04)$

$= 4.62$

Average number of failures per week

$$= \frac{1000}{4.62} = 216$$

- Cost of individual replacement = Rs 4 (216) = Rs 864/-
- (ii) Let N_i denote the total number of replacements made at the end of the i^{th} week

12

$N = 1000$

$N_1 = Np_1 = 1000\ (0.05) = 50$

$N_2 = Np_2 + N_1p_1$
$= 1000\ (0.08) + 50\ (0.05) - 82$

$N_3 = Np_3 + N_1p_2 + N_2p_1$
$= 1000\ (0.12) + 50\ (0.08) + 82\ (0.05)$
$= 128$

$N_4 = Np_4 + N_1p_3 + N_2p_2 + N_3p_1$
$= 1000\ (0.18) + 50\ (0.12) + 82\ (0.08) + 128\ (0.05)$
$= 199$

$N_5 = Np_5 + N_1p_4 + N_2p_3 + N_3p_2 + N_4p_1$
$= 1000\ (0.25) + 50\ (0.18) + 82\ (0.12) + 128\ (0.08) + 199\ (0.05)$
$= 289$

$N_6 = Np_6 + N_1p_5 + N_2p_4 + N_3p_3 + N_4p_2 + N_5p_1$
$= 1000\ (0.20) + 50\ (0.25) + 82\ (0.18) + 128\ (0.12) + 199\ (0.08) + 289\ (0.05)$
$= 272$

$N_7 = Np_7 + N_1p_6 + N_2p_5 + N_3p_4 + N_4p_3 + N_5p_2 + N_6p_1$
$= 1000\ (0.08) + 50\ (0.20) + 82\ (0.25) + 128\ (0.18) + 199\ (0.12) + 289\ (0.08) + 272\ (0.05)$
$= 194$

$N_8 = Np_8 + N_1p_7 + N_2p_6 + N_3p_5 + N_4p_4 + N_5p_3 + N_6p_2 + N_7p_1$
$= 1000\ (0.04) + 50\ (0.08) + 82\ (0.20) + 128\ (0.25) + 199\ (0.18) + 289\ (0.12) + 272\ (0.08) + 194\ (0.05)$
$= 195$

13

End of week	*Total cost of group replacement*	*Average cost per week*
1	$1000 + 50 \times 4 = 1200$	1200
2	$1000 + (50 + 82)4 = 1528$	764
3	$1000 + (50 + 82 + 128)4 = 2040$	680 ⟶
4	$1000 + (50 + 82 + 128 + 199)4 = 2836$	709
5	$1000 + (50 + 82 + 128 + 199 + 289)4 = 3992$	798
6	$1000 + (50 + 82 + 128 + 199 + 289 + 272)4 = 5080$	847

We find that the weekly average cost is least at the end of the 3rd week. Hence group replacement is to be made at the end of 3rd week.

Note: Comparing the two types of replacement we find that the total cost of individual replacement for 3 weeks, is $3 \times 864 = Rs\ 2592$.

Under group replacement the total cost of replacement for 3 weeks is Rs 2040 only. Hence group replacement at the end of every 3 weeks is less expensive than individual replacement.

14

Example 4

- The following table gives the probability distribution of the failure time of a machine:

Week	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
Prob. of failure	.03	.04	.05	.06	.07	.08	0.09	0.16	0.20	0.22

The cost of repairing a broken machine is Rs 200. Preventive maintenance service is done for all the 30 machines collectively at Rs 15 per machine at the end of a period T. Find T so as to minimize the cost of maintenance.

15

Solution

$N_1 = Np_1 = 30\ (0.03) = 0.9 \cong 1$

$N_2 = Np_2 + N_1p_1 = 30(04) + 1\ (0.03)$
$= 1.23 \cong 1$

Similarly,

$N_3 = 2$, $N_4 = 2$, $N_5 = 2$, $N_6 = 3$, $N_7 = 3$, $N_8 = 6$, $N_9 = 7$, $N_{10} = 8$

End of week	*Total cost of maintenance*	*Average cost*
1	$(30 \times 15) + (1)200 = 650$	650
2	$(30 \times 15) + (1 + 1)200 = 850$	425
3	$(30 \times 15) + (1 + 1 + 2)200 = 1250$	417
4	$(30 \times 15) + (1 + 1 + 2 + 2)200 = 1650$	412
5	$(30 \times 15) + (1 + 1 + 2 + 2 + 2)200 = 2050$	410 —
6	$(30 \times 15) + (1 + 1 + 2 + 2 + 2 + 3)200 = 2650$	442
7	$(30 \times 15) + (1 + 1 + 2 + 2 + 2 + 3 + 3)200 = 3250$	464
8	$(30 \times 15) + (1 + 1 + 2 + 2 + 2 + 3 + 3 + 6)200 = 4450$	556
9	$(30 \times 15) + (1 + 1 + 2 + 2 + 2 + 3 + 3 + 6 + 7)200 = 5850$	650
10	$(30 \times 15) + (1 + 1 + 2 + 2 + 2 + 3 + 3 + 6 + 7 + 8)200 = 7450$	745

The average cost is minimum at the end of 5 weeks.
Hence group service should be done at the end of the 5th week.

16