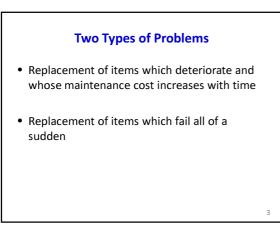
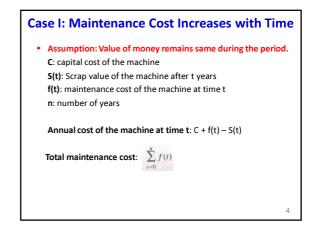
Replacement Problems

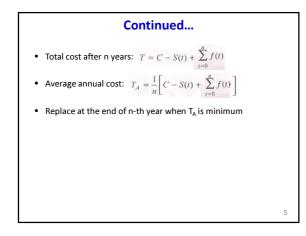
RK Jana

Introduction

- Machines, equipments, parts lose efficiency
- Planned replacement would reduce maintenance cost and other overhead expenses
- The problem is to find the age at which it is most economical to replace it
- Certain items/parts of items fail suddenly (radio, TV, bulb)
- Immediate replacement







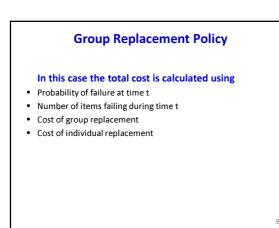
Example 1	scrap	valu	e costs Rs e is Rs 20	00. TI	ne mainte	enance	costs o
	the m	achir	ic are giv	en be	low:		
	Year	12	1 2	3	4 5	6	7 8
	Main nance		200 500	800	1200 1800	2500 3	200 4000
	W	hen s	hould the	macl	nine be r	eplaced	1?
C = 12,200							
S(t) = 200,	Year	f(t)	$\Sigma j(t)$	S(t)	C-S(t)	7	74
	1	200	200	200	12000	12200	12200
$T_A = \frac{C - S(t) + \Sigma f(t)}{T_A}$	2	500	700	200	12000	12700	6350
n n	3	800	1500	200	12000	13500	4500
	4	1200	2700	200	12000	14700	3673
	5	1800	4500	200	12000	16500	3300
	6	2500	7000	200	12000	19000	3167
	7	3200	10200	200	12000	22200	3171
	8	4000	14200	200	12000	26200	3275
T _A is minimum at the end of At the end of 6-th year	6-th yea	ar. Hen	ice it is pro	fitable	to replace	the ma	i chine

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Example 2		1			ck are	-				
	Year	2018	1	2	3	4	5	6	7	8
	Main nance Resal price	cost			1700 1200		2900 500	3800 400	4800 400	6000 400
	Tł	e pur	chas	e pr	ice o	f the	truc	k is	Rs	8000
C = 8000		mine		time	at v	which	ı it i	s pro		sle te
	Dete		the		at v	which	n it i	s pro		ole to
$T = C - S(t) + \Sigma f(t)$	Dete	mine	the trucl		at v		h it i $-S(t)$	1		T _a
	Deter	mine ce the	the trucl Σ	k.) C		1	ofital T	
$T = C - S(t) + \Sigma f(t)$ $T_{A} = \frac{T}{2}$	Deter	The ce the $f(t)$	the truck Σ	k. 7(1)	S(t) C	- S(t)		T 00	T _A
$T = C - S(t) + \Sigma f(t)$ $T_{A} = \frac{T}{2}$	Deter repla <u>Year</u> 1	$\frac{f(t)}{1000}$	the truck Σ 10 23	k. <i>7(1)</i> 000	<i>S(t</i> 4000) C	- <i>S(t)</i> 4000	500	<i>T</i> 00	T _A 5000
$T = C - S(t) + \Sigma f(t)$ $T_{4} = \frac{T}{-}$	Deter repla <u>Year</u> 1 2	$\frac{f(t)}{1000}$	the trucl Σ 10 23 40	k. 7(1) 000 500	S(t 4000 2000) C) 4) (- <i>S(t)</i> 1000 5000	500	<i>T</i> 00 00 00	T _A 5000 4150
$T = C - S(t) + \Sigma f(t)$ $T_A = \frac{T}{2}$	Deter repla Vear 1 2 3	rmine ce the <u>f(t)</u> 1000 1300 1700	the trucl 23 40 62	k. 7(1) 000 500	S(t) 4000 2000 1200) C) 4) 6) 6	- <i>S(t)</i> 1000 5000 5800	500 830 1080	7 00 00 00 00	T _A 5000 4150 3600
$T = C - S(t) + \Sigma f(t)$ $T_{A} = \frac{T}{-}$	Deter repla Vear 1 2 3 4	rmine ce the <i>f(t)</i> 1000 1300 1700 2200	the trucl 23 40 62	k. 7(1) 000 800 000 200 00	S(1 4000 2000 1200 600) C) 4) 6) 6) 7	- <i>S(t)</i> 4000 5000 5800 7400	500 830 1080 1360	7 00 00 00 00 00	T _A 5000 4150 3600 3400
$T = C - S(t) + \Sigma f(t)$ $T_{A} = \frac{T}{2}$	Deter repla <u>Year</u> 1 2 3 4 5	rmine ce the <u>f(t)</u> 1000 1300 1700 2200 2900	the truck 23 40 62 91	k. 7(t) 000 000 200 200 00	<i>S(t)</i> 4000 2000 1200 600 500) C 0 4 0 6 0 7 0 7 0 7	- <i>S(t)</i> 4000 5000 5800 7400 7500	500 830 1080 1360 1660	7 00 00 00 00 00 00 00	T ₄ 5000 4150 3600 3400 3320







E(x)

- p(x): probability that an item will fail at the age of x
- E(x): Average failure age
- N: Number of items in the group
- C₁ : per unit replacement cost
- C₂ : individual replacement cost
 - f(i): number of failure

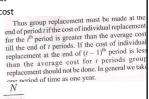
C(t): total cost

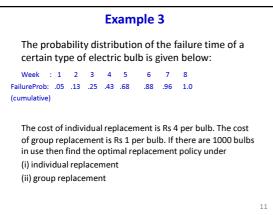
$$F(x) = \sum_{k=1}^{k} x p(x)$$

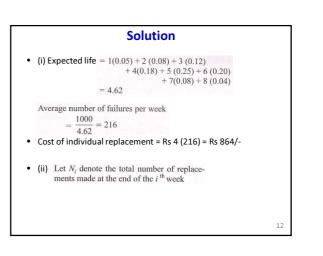
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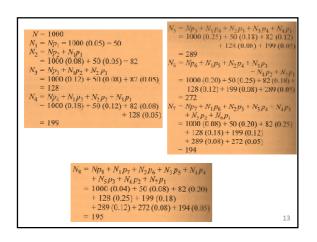
$$E(x) = \sum_{x=1}^{x} x^{p} (x)$$

Number of failure per unit









End of week	Total cost of group replacement	Average cost per week
1	$1000 + 50 \times 4 = 1200$	1200
2	1000 + (50 + 82)4 = 1528	764
3	1000 + (50 + 82 + 128)4 = 2040	$680 \longrightarrow$
4	1000 + (50 + 82 + 128 + 199)4 = 2836	709
5	1000 + (50 + 82 + 128 + 199 + 289)4 = 3992	798
6	1000 + (50 + 82 + 128 + 199 + 289 + 272)4 = 5080	847
	the end of the 3rd week. Hence group replacement	
	is to be made at the end of 3^{rd} week. Note: Comparing the two types of replacement we find that the total cost of individual replacement for 3 weeks, is $3 \times 864 = \text{Rs } 2592$. Under group replacement the total cost of replacement for 3 weeks is Rs 2040 only.	
	Note: Comparing the two types of replacement we find that the total cost of individual replacement for 3 weeks, is $3 \times 864 = \text{Rs} 2592$. Under group replacement the total cost of	

followir Ire time	-		-		he	pro	babi	lity o	distr	ibuti	on of
Week				4	5	6	7	8	9	10	
Prob. o failure	f .03	.04	.05	.06	.07	.08	0.09	0.16	0.20	0.22	
The Rs 200. for all t	Pre he 3	ver 0 n	ntiv nac ene	re n hin d of	nain es (f a j	nten coll	nanc lecti	e sei vely 7. Fi	rvico at F	e is a	done 5 per

	Solution	
$V_2 = .$	$Np_1 = 30 \ (0.03) = 0.9 \cong 1$ $Np_2 + N_1p_1 = 30(04) + 1 \ (0.03)$ $1.23 \cong 1$	
imil	arly,	
N. =	2_{1} , $N_{4} = 2_{2}$, $N_{5} = 2_{2}$, $N_{6} = 3_{2}$, $N_{7} = 3_{2}$, $N_{8} = 6_{2}$, $N_{9} = 7_{2}$, N_{10}	= 8
••3	-, ···4 2, ···5 2, ···6 3) ···7 3) ···8 3) ···9 7	, 0
1.0	Total cost of maintanance	rana cost
d of we		erage cost
1	$(30 \times 15) + (1)200 = 650$	650
1	$(30 \times 15) + (1)200 = 650 (30 \times 15) + (1 + 1)200 = 850$	650 425
1 2 3	$\begin{array}{c} (30 \times 15) + (1)200 = 650 \\ (30 \times 15) + (1+1)200 = 850 \\ (30 \times 15) + (1+1+2)200 = 1250 \end{array}$	650 425 417
1 2 3 4	$\begin{array}{c} (30 \times 15) + (1)200 = 650 \\ (30 \times 15) + (1 + 1)200 = 850 \\ (30 \times 15) + (1 + 1 + 2)200 = 1250 \\ (30 \times 15) + (1 + 1 + 2 + 2)200 = 1650 \end{array}$	650 425 417 412
1 2 3	$\begin{array}{c} (30\times15)+(1)200=650\\ (30\times15)+(1+1)200=850\\ (30\times15)+(1+1+2)200=1250\\ (30\times15)+(1+1+2)200=1250\\ (30\times15)+(1+1+2+2)200=1650\\ (30\times15)+(1+1+2+2+2)200=2050\\ \end{array}$	650 425 417 412 410-
4 5 6	$\begin{array}{c} (30 \times 15) + (1)200 = 650 \\ (30 \times 15) + (1 + 1)200 = 850 \\ (30 \times 15) + (1 + 1 + 2)200 = 1250 \\ (30 \times 15) + (1 + 1 + 2)200 = 1650 \\ (30 \times 15) + (1 + 1 + 2 + 2 + 2)200 = 2050 \\ (30 \times 15) + (1 + 1 + 2 + 2 + 2) + 3)200 = 2650 \end{array}$	650 425 417 412 410- 442
1 2 3 4 5 6 7 8	$\begin{array}{c} (30\times15)+(1)200=650\\ (30\times15)+(1+1)200=850\\ (30\times15)+(1+1+2)200=1250\\ (30\times15)+(1+1+2+2)200=1650\\ (30\times15)+(1+1+2+2+2)200=2050\\ (30\times15)+(1+1+2+2+2+3)200=2650\\ (30\times15)+(1+1+2+2+2+3+3)200=3250\\ \end{array}$	650 425 417 412 410- 442 464
1 2 3 4 5 6 7 8 9	$\begin{array}{c} (30\times15)+(1)200=650\\ (30\times15)+(1+1)200=850\\ (30\times15)+(1+1+2)200=1250\\ (30\times15)+(1+1+2)200=1250\\ (30\times15)+(1+1+2+2)200=1650\\ (30\times15)+(1+1+2+2+2+3)200=2650\\ (30\times15)+(1+1+2+2+2+3+3)200=3250\\ (30\times15)+(1+1+2+2+3+3+3)200=3250\\ (30\times15)+(1+1+2+3+3+3)200=3250\\ (30\times15)+(1+1+2+3+3+3)200=3250\\ (30\times15)+(1+1+2+3+3+3)200=3250\\ (30\times15)+(1+1+2+3+3+3+3)200=3250\\ (30\times15)+(1+1+2+3+3+3+3)200=3250\\ (30\times15)+(1+1+2+3+3+3+3)200=3250\\ (30\times15)+(1+1+2+3+3+3+3)200=3250\\ (30\times15)+(1+1+2+3+3+3+3)200=3250\\ (30\times15)+(1+1+2+3+3+3+3)200=3250\\ (30\times15)+(1+1+2+3+3+3+3)200=3250\\ (30\times15)+(1+1+2+3+3+3)200=3250\\ (30\times15)+(1+3+3+3)200=3250\\ (30\times15)+(1+3+3+3)200\\ (30\times15)+(1+3+3+3)200\\ (30\times15)+(1+3+3+3)200\\ (30\times15)+(1+3+3+3)200\\ (30\times15)+(1+3+3+3)20\\ (30\times15)+(1+3+3+3+3)20\\ (30\times15)+(1+3+3+3+3)20\\ (30\times15)+(1+3+3+3+3)20\\ (30\times15)+(1+3+3+3+3)20\\ (30\times15)+(1+3+3+3+3+3)20\\ (30\times15)+(1+3+3+3+3+3)20\\ (30\times15)+(1+3+3+3+3+3)20\\ (30\times15)+(1+3+3+3+3+3+3+3+3+3+3+3+3+3+3+3+3+3+3+$	650 425 417 412 410- 442 464 556
1 2 3 4 5 6 7 8 9	$\begin{array}{c} (30\times15)+(1)200=650\\ (30\times15)+(1+1)200=850\\ (30\times15)+(1+1+2)200=1250\\ (30\times15)+(1+1+2+2)200=1650\\ (30\times15)+(1+1+2+2+2)200=2050\\ (30\times15)+(1+1+2+2+2+3)200=2650\\ (30\times15)+(1+1+2+2+2+3+3)200=3250\\ \end{array}$	650 425 417 412 410- 442 464 556 650
1 2 3 4 5 6 7 8 9 10	$\begin{array}{c} (30\times15)+(1)200=650\\ (30\times15)+(1+1)200=850\\ (30\times15)+(1+1)2200=1250\\ (30\times15)+(1+1+2)2200=1250\\ (30\times15)+(1+1+2+2)200=2050\\ (30\times15)+(1+1+2+2+2)200=2050\\ (30\times15)+(1+1+2+2+2+3)200=2650\\ (30\times15)+(1+1+2+2+2+3+3)200=3250\\ (30\times15)+(1+1+2+2+2+3+3+6)200=4450\\ (30\times15)+(1+1+2+2+2+3+3+6)200=8580\\ \end{array}$	650 425 417 412 410- 442 464 556