Replacement Problems

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Introduction

• Machines, equipments, parts lose efficiency
• Planned replacement would reduce maintenance cost and other overhead expenses
• The problem is to find the age at which it is most economical to replace it
• Certain items/parts of items fail suddenly (radio, TV, bulb)
• Immediate replacement

Two Types of Problems

• Replacement of items which deteriorate and whose maintenance cost increases with time
• Replacement of items which fail all of a sudden

Case I: Maintenance Cost Increases with Time

• Assumption: Value of money remains same during the period.
• $C$: capital cost of the machine
• $S(t)$: Scrap value of the machine after $t$ years
• $f(t)$: maintenance cost of the machine at time $t$
• $n$: number of years

Annual cost of the machine at time $t$: $C + f(t) - S(t)$

Total maintenance cost: $\sum_{t=0}^{n} f(t)$

Continued...

• Total cost after $n$ years: $T = C - S(t) + \sum_{t=0}^{n} f(t)$
• Average annual cost: $T_a = \frac{1}{n} (C - S(t) + \sum_{t=0}^{n} f(t))$
• Replace at the end of $n$-th year when $T_a$ is minimum

Example 1

A machine costs Rs 12,000. The scrap value is Rs 300. The maintenance costs of the machine are given below:

<table>
<thead>
<tr>
<th>Year</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C$</td>
<td>12,000</td>
<td>12,000</td>
<td>12,000</td>
<td>12,000</td>
<td>12,000</td>
<td>12,000</td>
<td>12,000</td>
<td>12,000</td>
</tr>
<tr>
<td>$S(t)$</td>
<td>300</td>
<td>300</td>
<td>300</td>
<td>300</td>
<td>300</td>
<td>300</td>
<td>300</td>
<td>300</td>
</tr>
<tr>
<td>$f(t)$</td>
<td>200</td>
<td>200</td>
<td>200</td>
<td>200</td>
<td>200</td>
<td>200</td>
<td>200</td>
<td>200</td>
</tr>
<tr>
<td>$T_a$</td>
<td>12,000</td>
<td>12,000</td>
<td>12,000</td>
<td>12,000</td>
<td>12,000</td>
<td>12,000</td>
<td>12,000</td>
<td>12,000</td>
</tr>
</tbody>
</table>

When should the machine be replaced?

$T_a$ is minimum at the end of 6-th year. Hence it is profitable to replace the machine at the end of 6-th year.
Replacement of Items that Fail Completely

- There are many situations in which items do not deteriorate with time but fail all on a sudden completely. It may not be possible to predict the time of failure. Hence we make use of the probability distribution of the failure time which is obtained from past experience.
- In such situations, following two types of replacement policies are used:
  - Individual replacement policy (item replaced immediately after its failure)
  - Group replacement policy (all items are collectively replaced after a specific time period irrespective of whether they have failed or not)

Group Replacement Policy

In this case the total cost is calculated using

- Probability of failure at time t
- Number of items failing during time t
- Cost of group replacement
- Cost of individual replacement

Example 2

The maintenance cost and road price of a truck are given below:

<table>
<thead>
<tr>
<th>Year</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maint. cost</td>
<td>1000</td>
<td>1300</td>
<td>1500</td>
<td>2200</td>
<td>2600</td>
<td>3000</td>
<td>3800</td>
</tr>
<tr>
<td>Road price</td>
<td>4000</td>
<td>5000</td>
<td>4000</td>
<td>5000</td>
<td>4000</td>
<td>4000</td>
<td>4000</td>
</tr>
</tbody>
</table>

The purchase price of the truck is Rs 8000. Determine the time at which it is profitable to replace the truck.

\[ T = \frac{C}{T} \]

Example 3

The probability distribution of the failure time of a certain type of electric bulb is given below:

<table>
<thead>
<tr>
<th>Week</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>FailureProb</td>
<td>.05</td>
<td>.13</td>
<td>.25</td>
<td>.43</td>
<td>.68</td>
<td>.88</td>
<td>.96</td>
<td>1.0</td>
</tr>
</tbody>
</table>

The cost of individual replacement is Rs 4 per bulb. The cost of group replacement is Rs 1 per bulb. If there are 1000 bulbs in use then find the optimal replacement policy under

(i) Individual replacement
(ii) Group replacement

Notations

- \( p(x) \): probability that an item will fail at the age of x
- \( E(x) \): Average failure age
- \( N \): Number of items in the group
- \( C_1 \): per unit replacement cost
- \( C_2 \): individual replacement cost
- \( f(i) \): number of failure
- \( C(t) \): total cost

\[ C(t) = \sum_{x=1}^{t} x p(x) \]

Solution

(i) Expected life

\[ \text{Average number of failures per week} = \frac{1000}{4.62} = 216 \]

(ii) Cost of individual replacement = Rs 4 \times 216 = Rs 864 /

(iii) Let \( N_t \) denote the total number of replacements made at the end of the \( t^{th} \) week
Example 4

The following table gives the probability distribution of the failure time of a machine:

<table>
<thead>
<tr>
<th>Week</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>f(x)</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
</tr>
</tbody>
</table>

The cost of repairing a broken machine is Rs 200. Preventive maintenance service is done for all the 30 machines collectively at Rs 15 per machine at the end of a period T. Find T so as to minimize the cost of maintenance.

Solution

\[ N_1 = N_p = 30 \times 0.03 = 0.9 \approx 1 \]

Similarly,

\[ N_2 = N_3 = 2, N_4 = 2, N_5 = 3, N_6 = 3, N_7 = 7, N_8 = 7, N_9 = 7, N_{10} = 8 \]

We find that the weekly average cost is least at the end of the 3rd week. Hence group replacement is to be made at the end of the 3rd week.

Note: Comparing the two types of replacement, we find that the total cost of individual replacement for 3 weeks is Rs 2940 only.

Under group replacement, the total cost of replacement for 3 weeks is Rs 2340 only. Hence group replacement at the end of every 3 weeks is less expensive than individual replacement.