Decision Theory

RK Jana

Topics

- Introduction
- Payoff matrix
- Regret matrix
- Decision making situations
- Methods of different decision making situations
  - Decision making under uncertainty
  - Decision making under risk
- Decision Trees

Introduction

- External environment imposes certain restrictions on us. Based on our response to such restrictions, we get different payoffs.
- We can't change the payoff matrix.
- We can use the available information wisely to arrive at optimal decision.

An Example

A fruit seller buys strawberries at Rs 20/- per kg and sells them at Rs 50/- per kg. The product is perishable by nature and can't be stored overnight. It has to be sold on the day of purchase. From past experience, the seller knows that the daily demand will range between 10 to 13 kg. Every kg of strawberries brought and not sold would lead to a loss of Rs 20/-, while every kg of that could not be sold because of stock out would lead to an opportunity loss of Rs 30/- per kg.

The Payoff Matrix

		Possible stock allocation			
		10	11	12	13
Possible Demand (kg)	10	300	280	260	240
	11	300	330	310	290
	12	300	330	360	340
	13	300	330	360	390

The Regret Matrix

		Possible stock allocation			
		10	11	12	13
Possible Demand (kg)	10	0	20	40	60
	11	30	0	20	40
	12	60	30	0	20
	13	90	60	30	0

The Decision Making Process

- Determine various alternative courses of action from which the final decision is to be made.
- Indentify the possible outcomes, called the events or states of nature.
- Determine the payoff associated with each alternative for every future condition.
- Evaluate alternatives according to some decision criterion and select the best alternative.
- Construct the regret or opportunity loss.

7

Decision Alternatives & States of Nature

- The decision alternatives are the different possible strategies the decision maker can employ.
- The states of nature refer to future events, not under the control of the decision maker, which will ultimately affect decision results. States of nature should be defined so that they are mutually exclusive and contain all possible future events that could affect the results of all potential decisions.

8

Why We Make Poor Decisions?

- Availability of information
- Making caused by costs
- Human abilities
- Time
- Technology
- Sub-optimization (each attempting to reach a solution that is optimum for him/her)

9

Decision Environments

- Decisions under Certainty - Environment in which relevant parameters have known values (LPP, Transportation, Assignment etc).
- Decisions under Risk - Environment in which certain future events have probable outcomes (probabilities of occurrences can be obtained from past data).

10

Continued…

- Decisions under Uncertainty - Environment in which it is impossible to assess the likelihood of various future events.
- Decisions under Conflict - Environment in which partial information are available (two or more competitors marketing the same product).

11

Decisions under Certainty

In this situation, only payoffs are known and nothing is known about the likelihood of each state of nature. Popular methods are:

- Maximin (minimax) Criterion - Choose the alternative with the best (worst) of the worst (best) possible payoffs.
- Maximax (minimin) Criterion - Choose the alternative with the best (worst) possible payoffs.

12

Continued…

- Laplace Criterion - Choose the alternative with the best average payoff of any of the alternatives.
- Minimax Regret / Savage Criterion - Choose the alternative that has the least of the worst regrets.
- Hurwicz Criterion – Chooses the convex combination of the best and worst payoffs.

The Maximin Criterion

In this criterion, it is assumed that the worst possible is going to happen. The decision maker maximizes his minimum payoffs.

- **Step 1:** Determine the minimum assured payoffs for each alternative.
- **Step 2:** Choose the alternative which corresponds to the maximum of the above minimum payoffs.

The Minimax Criterion

In this criterion, it is assumed that the best possible is going to happen. The decision maker minimizes his maximum payoffs.

- **Step 1:** Determine the maximum assured payoffs for each alternative.
- **Step 2:** Choose the alternative which corresponds to the minimum of the above maximum payoffs.

Example 1

A businessman has three alternatives each of which can be followed by four possible events. The payoff matrix of which is as follows:

Alternatives	Payoffs			
↓	A	B	C	D
X	8	0	-10	6
Y	-4	12	18	-2
Z	14	6	0	8

Determine which alternative should he choose if he adopts

(i) maximin criterion

(ii) minimax criterion

Solution 1

(i) Maximin:

Minimum assured payoffs for each alternative –

X: - 10, Y: - 4, Z: 0

Max{ -10, -4, 0} = 0. So, alternative Z is selected.

(ii) Minimax:

Maximum assured payoffs for each alternative –

A: 14, B: 12, C: 18, D: 8

Min{ 14, 12, 18, 8} = 8. So, alternative D is selected.

Minimax Regret / Savage Criterion

The Savage criterion is based on the concept of regret or opportunity loss and the alternative corresponding to minimum of the maximum regrets is selected. The method is as follows:

- **Step 1:** Determine the amount of regret for payoff of each alternative for a particular event. The regret of *i*-th alternative when event *j* occurs is given by
  - A. For Profit: (Maximum payoffs – *i*-th payoff)
  - B. For Cost/Loss: (*i*-th payoff - Maximum payoffs)
- **Step 2:** Determine the maximum regret amount for each alternative.
- **Step 3:** Choose the alternative corresponding to the minimum regret.

Example 2

Alternatives ↓	Payoffs			
	A	B	C	D
X	8	0	-10	6
Y	-4	12	18	-2
Z	14	6	0	8

Alternatives ↓	Regret Payoffs				Maximum Regret
	A	B	C	D	
X	6	12	28	2	28
Y	18	0	0	10	18
Z	0	6	18	0	18
Max Payoff	14	12	18	8	Min = 18

Alternative Y or Z is selected as both correspond to minimum of the maximum possible regrets 18.

The Laplace Criterion

The Laplace criterion uses all information by assigning equal probabilities to the possible payoffs for each action and then selecting the alternative that corresponds to the maximum expected payoffs.

- **Step 1:** Assign equal probabilities (1/n, if there are n possible payoffs).
- **Step 2:** Determine the expected payoff for each alternative.
- **Step 3:** Select the alternative that corresponds to the maximum of the expected payoffs.

Example 3

Alternatives ↓	Payoffs			
	A	B	C	D
X	8	0	-10	6
Y	-4	12	18	-2
Z	14	6	0	8

- E(X) = (8+0-10+6)/4 = 1
- E(Y) = (-4+12+18-2)/4 = 6
- E(Z) = (14+6+0+8)/4 = 7

So, alternative Z is selected.

Hurwicz Criterion

This criterion specifies that a decision maker's view will fall between the extreme optimism of the maximum criterion and the extreme pessimism of the minimum criterion.

- **Step 1:** Choose an appropriate degree of optimism α ($0 \le \alpha \le 1$). So, $(1 - \alpha)$ is the degree of pessimism.
- **Step 2:** Determine the maximum as well as minimum payoffs for each alternative and obtain.
- **Step 3:** Calculate: α.(maximum) + (1- α).(minimum).

Example 4

Alternatives	Max Payoffs (MaxP)	Min Payoffs (MinP)	α	α.(MaxP) + (1-α).(MinP)
A	14	-4	0.5	5
B	12	0		6
C	18	-10		4
D	8	-2		3

The optimal solution is to choose D.

Decisions Under Risk

A decision maker is said to take decision under risk when he selects the alternative from several options available to him whose probabilities of occurrence can be stated. Important decision making under risk criteria are as follows:

- Expected Monetary Value (EMV) criterion
- Expected Opportunity Loss (EOL) criterion
- Expected Value of Perfect Information (EVPI) criterion

Expected Monetary Value (EMV) Criterion

EMV for a given course of action is the expected value of conditional payoffs for that action. It is summarized as follows:

- **Step 1:** List conditional profit for each act-event combination along with corresponding event probabilities.
- **Step 2:** For each act, determine the expected conditional profits.
- **Step 3:** Determine the EMV for each act.
- **Step 4:** Choose the act corresponding to the optimal EMV.

25

Example 6

A man has the choice of running either a hot-snack stall or an ice cream stall at a seaside resort during the summer. If it is a fairly cool summer, he should make a profit of Rs 5000/- by running the hot-snack stall whereas he should make Rs 1000/- if the summer is quite hot. On the other hand, his estimated profit is Rs 6500/- if he runs an ice cream stall in the hot summer and Rs 1000/- if the summer is cool. There is a 40% chance of the summer being hot. Should the man opt for running the hot snack stall or the ice cream stall?

26

Solution 6

E_1 : event that the summer is cool

E_2 : event that the summer is hot

		Conditional Payoffs		Expected Payoffs	
Event (E_i)	P(E_i)	HS Stall	IC Stall	HS Stall	IC Stall
Cool Summer	0.6	5000	1000	0.6×5000 = 3000	0.6×1000 =600
Hot Summer	0.4	1000	6500	0.4×1000 =400	0.4×6500 =2600

EMV from HS Stall = Rs(3000 + 400) = Rs 3400

EMV from IC Stall = Rs(600 + 2600) = Rs 3200

So, the man should opt for running the HS Stall.

27

Expected Opportunity Loss (EOL)

EOL is the alternative approach to EMV. In this case the EOL are calculated in a similar manner. The steps are as follows:

- **Step 1:** Construct the conditional profit table.
- **Step 2:** For each event, determine the conditional opportunity loss by first selection the maximum payoffs, and then finding the difference between the maximum payoffs and each conditional profit for that event.
- **Step 3:** For each act, determine the expected conditional opportunity losses and sum them to get the EOL.
- **Step 4:** Choose the act having minimum EOL.

28

Example 7

		Conditional Payoffs		Max Possible profit
Event (E_i)	P(E_i)	HS Stall	IC Stall	
Cool Summer	0.6	5000	1000	5000
Hot Summer	0.4	1000	6500	6500

		Opportunity Loss		Expected Losses	
Event (E_i)	P(E_i)	HS Stall	IC Stall	HS Stall	IC Stall
Cool Summer	0.6	0	4000	0.6×0 =0	0.6×4000 =2400
Hot Summer	0.4	5500	0	0.4×5500 =2200	0.4×0 =0

EOL for HS Stall = Rs(0 + 2200) = Rs 2200

EOL for IC Stall = Rs(2400 + 0) = Rs 2400

So, the man should opt for running the HS Stall as the loss in minimum for HS Stall.

29